

The Biases of Marginal Effect Estimators in Log-Transformed Regression Models

David E. Giles

Department of Economics

University of Victoria

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Abstract

Accurate estimation of marginal effects is of considerable interest to economists. We use “small disturbance” asymptotics to obtain analytic expressions for the biases of marginal effect estimators in regression models with a logarithmically transformed dependent variable, and regressors which may be in the levels or logarithms of the variables. Some numerical evaluations suggest that the relative magnitudes of these biases are small, but not as small as their counterparts for elasticity estimators.

Key Words: Bias, small-disturbance asymptotics, marginal effects, logarithmic regression

JEL Classification Codes: C2, C13

Contact information: David E. Giles, Department of Economics, University of Victoria, P.O. Box 1700 STN CSC, Victoria, B.C., Canada V8W2Y2; dgiles@uvic.ca

1. Introduction

The concept of a “marginal effect” is central in economics. It measures the change in a response variable resulting from a unit change in an explanatory variable. If the relationship between these variables is measured by a regression model that is linear in the parameters and the variables, the marginal effects are the regression coefficients themselves. Accordingly, the statistical properties of the estimated marginal effects are just those of the estimated regression coefficients. For example, the estimator of a marginal effect will be best linear unbiased if the estimator of the corresponding coefficient has these properties.

This situation changes if the model is non-linear, in either the parameters or the variables. In such models, the estimators of the marginal effects are typically non-linear functions of the random sample data and/or the parameters. These estimators are then biased, in general, even if they are constructed using an unbiased estimator of the coefficients themselves.

This basic point is generally ignored by applied researchers. In this paper we re-consider the expressions for the marginal effects in regression models that are linear in the parameters, but have a dependent variable that has been log-transformed¹. The explanatory variables may or may not be log-transformed². Models of this type are widely used. We derive analytic expressions for the biases of marginal effect estimators for such models. These expressions are based on "small-disturbance" (“small-sigma”) approximations, so they are valid for any sample size.

The next section introduces the model and notation, and provides some preliminary results. The bias expressions for the marginal effect estimators are derived in section 3; and these are illustrated numerically in section 4. Section 5 concludes.

2. Preliminary results

The model we consider is

$$z = \log(y) = X\beta + \sigma u . \tag{1}$$

The $(n \times p)$ matrix, X , is non-random and has full rank, p .

The i^{th} row and j^{th} column of X will be denoted by x_i' and x_j respectively. So, the i^{th} observation on the model can be written as

$$z_i = \log(y_i) = x_i'\beta + \sigma u_i \quad ; \quad i = 1, 2, \dots, n. \tag{2}$$

Let X_{-j} be the $[n \times (p - 1)]$ matrix formed by deleting x_j from X . Let M_{-j} be the idempotent matrix, $I_n - X_{-j}(X_{-j}'X_{-j})^{-1}X_{-j}$. Note that $M_{-j}X\beta = M_{-j}x_j\beta_j$.

Equation (2) takes the form

$$z_i = \mu + \sigma u_i ; \quad i = 1, 2, 3, \dots, n \quad (3)$$

where $\mu \neq 0$ and we assume that the u_i 's are independently and identically distributed, with

$$E[u_i] = 0 ; E[u_i^2] = 1 ; E[u_i^3] = \gamma_1 ; E[u_i^4] = \gamma_2 + 3. \quad (4)$$

So, the distribution of the u_i 's has a skewness coefficient of γ_1 and excess kurtosis of γ_2 .

If the j^{th} regressor in (2) is measured in levels, then the i^{th} observation on the marginal effect associated with that regressor is

$$m_{ij}^{(1)} = \beta_j y_i = \beta_j \exp(z_i). \quad (5)$$

We refer to this as the “log-lin” model. The “log-log” model arises when the j^{th} regressor in (2) is log-transformed. In this case, the i^{th} observation on the marginal effect associated with that regressor is

$$m_{ij}^{(2)} = \left[\frac{\beta_j y_i}{w_{ij}} \right] = \beta_j \exp(z_i - x_{ij}), \quad (6)$$

where $x_{ij} = \log(w_{ij})$, say, is the i^{th} observation on the regressor of interest.

The marginal effects in (5) and (6) are non-linear functions of the random z data. This non-linearity is complicated further when β_j is estimated. Even if an unbiased estimator (such as OLS) of this parameter is used, the marginal effect estimators themselves are biased. Although exact bias expressions are intractable, various approximations may be considered. Large- n asymptotic expansions (*e.g.*, Edgeworth expansions) could be used, but these are based on strong moment conditions that may be difficult to verify, and they are of limited use if the sample size is small.

Alternatively, a small- σ expansion of the type suggested by Kadane (1971) may be used. A small-disturbance approximation has increasing accuracy as the precision of the model increases, and has proved to be extremely useful in a variety of econometric problems. For example, see Chen and Giles (2007), Inder (1989), King and Wu (1991), and Peters (1989). An overview is provided by Ullah (2004, pp. 36-45).

To obtain our small- σ expressions for the biases of the marginal effect estimators, to $O(\sigma^4)$, we use the following result from Ullah *et al.* (1995):

Lemma

Let z be an n -element random vector, with $z = \mu + \sigma u$, where the elements of u satisfy the conditions in (4) and the non-zero mean, μ , is a function of a parameter vector, θ . Let $\hat{\theta} = h(z)$ be an estimator of θ , where $h(z)$ and its derivatives exist in a neighborhood of μ . Then

$$E(\hat{\theta} - \theta) = \sigma^2 \Delta_2 + \sigma^3 \gamma_1 \Delta_3 + \sigma^4 (\gamma_2 \Delta_4 + 3\Delta_{22}) , \quad (7)$$

where, for $s = 2, 3, 4$:

$$\Delta_s = \frac{1}{s!} \sum_{k=1}^n \left[\frac{\partial^s h(z)}{\partial z_k^s} \right]_{z=\mu} ; \quad \Delta_{22} = \frac{1}{4!} \sum_{k=1}^n \sum_{l=1}^n \left[\frac{\partial^4 h(z)}{\partial z_k^2 \partial z_l^2} \right]_{z=\mu} .$$

3. Bias expressions

3.1 Marginal effects at a single observation

Consider the log-lin model. The estimator for the marginal effect for the j^{th} regressor at the i^{th} observation is $\hat{m}_{ij}^{(1)} = b_j \exp(z_i)$, where b_j is the OLS estimator of the j^{th} regression coefficient, and z_i is the i^{th} observation on the dependent variable. Notice that \hat{m}_{ij} is a non-linear function of the random z data. Using standard partitioned regression results, we have:

$$\hat{m}_{ij}^{(1)} = (x_{.j}' M_{-j} x_{.j})^{-1} (x_{.j}' M_{-j} z) \exp(z_i) = (x_{.j}' M_{-j} x_{.j})^{-1} h(z) , \quad (8)$$

where $z_i = \log(y_i)$,

$$h(z) = (x_{.j}' M_{-j} z) \exp(z_i) = (x_{.j}' M_{-j} z^*) , \quad (9)$$

and

$$z^* = z \exp(z_i) . \quad (10)$$

For the log-log model, the j^{th} regressor is $x'_{.j} = \log(w'_{.j})$. The corresponding marginal effect estimator is:

$$\hat{m}_{ij}^{(2)} = b_j (\exp(z_i) / w_{ij}) = [w_{ij} (x_{.j}' M_{-j} x_{.j})]^{-1} h(z) , \quad (11)$$

where $h(z)$ is defined in (9), and w_{ij} is the i^{th} element of w_j' .

The biases of the estimators in (8) and (11) are:

$$\text{Bias} \left[\widehat{m}_{ij}^{(1)} \right] = (x_{.j}' M_{-j} x_{.j})^{-1} \text{Bias}[h(z)] ; \quad (12)$$

and

$$\text{Bias} \left[\widehat{m}_{ij}^{(2)} \right] = [w_{ij} (x_{.j}' M_{-j} x_{.j})]^{-1} \text{Bias}[h(z)] . \quad (13)$$

So, we need to obtain a small-disturbance approximation for $\text{Bias}[h(z)]$.

From (9), and applying the Lemma:

Theorem 1

To $O(\sigma^4)$, $\text{Bias} \left[\widehat{m}_{ij}^{(1)} \right]$ is the same for all sample observations, i , and

$$\begin{aligned} \text{Bias} \left[\widehat{m}_{ij}^{(1)} \right] &= \frac{\sigma^2}{2!} \left[\beta_j \sum_{k=1}^n \exp(x_k' \beta) + 2(x_{.j}' M_{-j} x_{.j})^{-1} \sum_{k=1}^n \exp(x_k' \beta) (x_{.j}' M_{-j} t_k) \right] \\ &\quad + \frac{\sigma^3 \gamma_1}{3!} \left[\beta_j \sum_{k=1}^n \exp(x_k' \beta) + 3(x_{.j}' M_{-j} x_{.j})^{-1} \sum_{k=1}^n \exp(x_k' \beta) (x_{.j}' M_{-j} t_k) \right] \\ &\quad + \frac{\sigma^4 (\gamma_2 + 3)}{4!} \left[\beta_j \sum_{k=1}^n \exp(x_k' \beta) + 4(x_{.j}' M_{-j} x_{.j})^{-1} \sum_{k=1}^n \exp(x_k' \beta) (x_{.j}' M_{-j} t_k) \right] \end{aligned}$$

Proof

From (9) and (10), for $s = 2, 3, 4$,

$$\frac{\partial^s h(z)}{\partial z_k^s} = \exp(z_i) x_{.j}' M_{-j} (z + s t_k) ; \quad (k = i) \quad (14)$$

$$= 0 ; \quad (k \neq i)$$

where t_k is an $(n \times 1)$ vector with k^{th} element equal to unity, and all other elements equal to zero.

Similarly,

$$\frac{\partial^4 h(z)}{\partial z_k^2 \partial z_l^2} = \exp(z_i) x_{.j}' M_{-j} (z + 4 t_k) ; \quad (k = l = i) \quad (15)$$

$$= 0 ; \quad (\text{otherwise})$$

Recalling that $E(z_i) = x_i' \beta$, $E(z) = X\beta$, and $M_{-j}X\beta = M_{-j}x_j\beta_j$, we have:

$$\Delta_s = \frac{1}{s!} \left[(x_j' M_{-j} x_j) \beta_j \sum_{k=1}^n \exp(x_k' \beta) + s \sum_{k=1}^n \exp(x_k' \beta) (x_j' M_{-j} t_k) \right] ; \quad s = 2, 3, 4 \quad (16)$$

and $\Delta_{22} = \Delta_4$.

Substituting these expressions into (7), and recalling that $Bias[\hat{m}_j^{(1)}] = (x_j' M_{-j} x_j)^{-1} Bias[h(z)]$, the result follows directly. \square

If the regression model includes a single regressor and no intercept, the result in Theorem 1 simplifies:

Corollary 1

When the log-lin model contains just a single regressor, and no intercept, so that $z_i = \log(y_i) = \beta x_i + \sigma u_i$; $i = 1, 2, 3, \dots, n$; then the bias of the marginal effect, to $O(\sigma^4)$, is:

$$\begin{aligned} Bias[\hat{m}^{(1)}] &= \frac{\sigma^2}{2!} \left[\beta \sum_{k=1}^n \exp(\beta x_k) + (2 / \sum_{j=1}^n x_j^2) \sum_{k=1}^n x_k \exp(\beta x_k) \right] \\ &\quad + \frac{\sigma^3 \gamma_1}{3!} \left[\beta \sum_{k=1}^n \exp(\beta x_k) + (3 / \sum_{j=1}^n x_j^2) \sum_{k=1}^n x_k \exp(\beta x_k) \right] \\ &\quad + \frac{\sigma^4 (\gamma_2 + 3)}{4!} \left[\beta \sum_{k=1}^n \exp(\beta x_k) + (4 / \sum_{j=1}^n x_j^2) \sum_{k=1}^n x_k \exp(\beta x_k) \right] \end{aligned}$$

The bias is easily signed in this case. If all of the x_i 's and β are of the same sign, the bias to $O(\sigma^2)$ is positive. If all of the x_i 's are of the same sign but β has the opposite sign, the bias to $O(\sigma^2)$ is negative. These two results also hold to $O(\sigma^3)$ if the disturbances' distribution is symmetric or positively skewed. They are reversed in sign if that distribution is negatively skewed. Recalling that $\gamma_2 \geq -2$ for all distributions, these results to $O(\sigma^3)$ also hold to $O(\sigma^4)$.

In the case of the log-log model, we have:

Theorem 2

To $O(\sigma^4)$,

$$\begin{aligned} \text{Bias} [\hat{m}_{ij}^{(2)}] &= \frac{\sigma^2}{w_{ij}2!} \left[\beta_j \sum_{k=1}^n \exp(x_k' \beta) + 2(x_j' M_{-j} x_j)^{-1} \sum_{k=1}^n \exp(x_k' \beta) (x_j' M_{-j} t_k) \right] \\ &+ \frac{\sigma^3 \gamma_1}{w_{ij}3!} \left[\beta_j \sum_{k=1}^n \exp(x_k' \beta) + 3(x_j' M_{-j} x_j)^{-1} \sum_{k=1}^n \exp(x_k' \beta) (x_j' M_{-j} t_k) \right] \\ &+ \frac{\sigma^4 (\gamma_2 + 3)}{w_{ij}4!} \left[\beta_j \sum_{k=1}^n \exp(x_k' \beta) + 4(x_j' M_{-j} x_j)^{-1} \sum_{k=1}^n \exp(x_k' \beta) (x_j' M_{-j} t_k) \right] \end{aligned}$$

The proof follows that of Theorem 1, noting that $[\hat{m}_{ij}^{(2)}] = [w_{ij}(x_j' M_{-j} x_j)]^{-1} \text{Bias}[h(z)]$.

Here, the bias of the j^{th} marginal effect varies according to the value of the sample observations.

Corollary 2

When the log-log model contains just a single regressor, and no intercept, the bias of the marginal effect, to $O(\sigma^4)$, is:

$$\begin{aligned} \text{Bias} [\hat{m}_i^{(2)}] &= \frac{\sigma^2}{w_i 2!} \left[\beta \sum_{k=1}^n \exp(\beta x_k) + (2 / \sum_{j=1}^n x_j^2) \sum_{k=1}^n x_k \exp(\beta x_k) \right] \\ &+ \frac{\sigma^3 \gamma_1}{w_i 3!} \left[\beta \sum_{k=1}^n \exp(\beta x_k) + (3 / \sum_{j=1}^n x_j^2) \sum_{k=1}^n x_k \exp(\beta x_k) \right] \\ &+ \frac{\sigma^4 (\gamma_2 + 3)}{w_i 4!} \left[\beta \sum_{k=1}^n \exp(\beta x_k) + (4 / \sum_{j=1}^n x_j^2) \sum_{k=1}^n x_k \exp(\beta x_k) \right] \end{aligned}$$

The log-log model is defined only if $w_i > 0$, for all i . The signing of the bias here is identical to that for Corollary 1.

3.2 Marginal effects at the sample average

The marginal effect at the sample average for the log-lin model is

$$m_j^{(1)} = \beta_j \bar{y} = (\beta_j / n) \sum_{i=1}^n \exp(z_i). \quad (17)$$

That for the log-log model is

$$m_j^{(2)} = \left[\frac{\beta_j \bar{y}}{\bar{w}} \right] = \beta_j \left[\sum_{i=1}^n \exp(z_i) \right] / (n\bar{w}) . \quad (18)$$

As before, $x_{ij} = \log(w_{ij})$ is the i^{th} observation on the regressor of interest.

The OLS estimators for these marginal effects are

$$\hat{m}_j^{(1)} = (nx_{.j}'M_{-j}x_{.j})^{-1} (x_{.j}'M_{-j}z) \sum_{i=1}^n \exp(z_i) = (nx_{.j}'M_{-j}x_{.j})^{-1} f(z) , \quad (19)$$

and

$$\hat{m}_j^{(2)} = [n\bar{w}(x_{.j}'M_{-j}x_{.j})]^{-1} f(z) , \quad (20)$$

where

$$f(z) = (x_{.j}'M_{-j}z) \sum_{i=1}^n \exp(z_i) = (x_{.j}'M_{-j}z^{**}) , \quad (21)$$

and

$$z^{**} = z \sum_{i=1}^n \exp(z_i) . \quad (22)$$

From (21) and (22) we have

$$\frac{\partial^s f(z)}{\partial z_k^s} = \exp(z_k) x_{.j}' M_{-j} (z + s \iota_k) \quad ; \quad s = 2, 3, 4 \quad (23)$$

$$\frac{\partial^4 h(z)}{\partial z_k^2 \partial z_l^2} = \exp(z_k) x_{.j}' M_{-j} (z + 4 \iota_k) \quad ; \quad k = l \quad (24)$$

$$= 0 \quad ; \quad (\text{otherwise})$$

Equations (22) and (23) are identical to (14) and (15), so the following results come immediately from (19).

Theorem 3

For the log-lin model the bias of the j^{th} marginal effect, evaluated at the sample mean of the data is, to $O(\sigma^4)$,

$$\begin{aligned}
Bias[\hat{m}_j^{(1)}] &= \frac{\sigma^2}{n2!} \left[\beta_j \sum_{k=1}^n \exp(x_k' \beta) + 2(x_j' M_{-j} x_j)^{-1} \sum_{k=1}^n \exp(x_k' \beta) (x_j' M_{-j} t_k) \right] \\
&+ \frac{\sigma^3 \gamma_1}{n3!} \left[\beta_j \sum_{k=1}^n \exp(x_k' \beta) + 3(x_j' M_{-j} x_j)^{-1} \sum_{k=1}^n \exp(x_k' \beta) (x_j' M_{-j} t_k) \right] \\
&+ \frac{\sigma^4 (\gamma_2 + 3)}{n4!} \left[\beta_j \sum_{k=1}^n \exp(x_k' \beta) + 4(x_j' M_{-j} x_j)^{-1} \sum_{k=1}^n \exp(x_k' \beta) (x_j' M_{-j} t_k) \right]
\end{aligned}$$

Corollary 3

When the log-lin model contains just a single regressor and no intercept, the bias of the marginal effect, evaluated at the sample mean of the data is, to $O(\sigma^4)$:

$$\begin{aligned}
Bias[\hat{m}^{(1)}] &= \frac{\sigma^2}{n2!} \left[\beta \sum_{k=1}^n \exp(\beta x_k) + (2 / \sum_{j=1}^n x_j^2) \sum_{k=1}^n x_k \exp(\beta x_k) \right] \\
&+ \frac{\sigma^3 \gamma_1}{n3!} \left[\beta \sum_{k=1}^n \exp(\beta x_k) + (3 / \sum_{j=1}^n x_j^2) \sum_{k=1}^n x_k \exp(\beta x_k) \right] \\
&+ \frac{\sigma^4 (\gamma_2 + 3)}{n4!} \left[\beta \sum_{k=1}^n \exp(\beta x_k) + (4 / \sum_{j=1}^n x_j^2) \sum_{k=1}^n x_k \exp(\beta x_k) \right]
\end{aligned}$$

Similarly, from (20), we have:

Theorem 4

For the log-log model the bias of the j^{th} marginal effect, evaluated at the sample mean of the data is, to $O(\sigma^4)$,

$$\begin{aligned}
Bias[\hat{m}_j^{(2)}] &= \frac{\sigma^2}{n\bar{w}_j 2!} \left[\beta_j \sum_{k=1}^n \exp(x_k' \beta) + 2(x_j' M_{-j} x_j)^{-1} \sum_{k=1}^n \exp(x_k' \beta) (x_j' M_{-j} t_k) \right] \\
&+ \frac{\sigma^3 \gamma_1}{n\bar{w}_j 3!} \left[\beta_j \sum_{k=1}^n \exp(x_k' \beta) + 3(x_j' M_{-j} x_j)^{-1} \sum_{k=1}^n \exp(x_k' \beta) (x_j' M_{-j} t_k) \right] \\
&+ \frac{\sigma^4 (\gamma_2 + 3)}{n\bar{w}_j 4!} \left[\beta_j \sum_{k=1}^n \exp(x_k' \beta) + 4(x_j' M_{-j} x_j)^{-1} \sum_{k=1}^n \exp(x_k' \beta) (x_j' M_{-j} t_k) \right]
\end{aligned}$$

Corollary 4

When the log-log model contains just a single regressor and no intercept, the bias of the marginal effect, evaluated at the sample means of the data is, to $O(\sigma^4)$:

$$\begin{aligned}
Bias[\hat{m}^{(2)}] &= \frac{\sigma^2}{n\bar{w}2!} \left[\beta \sum_{k=1}^n \exp(\beta x_k) + (2/\sum_{j=1}^n x_j^2) \sum_{k=1}^n x_k \exp(\beta x_k) \right] \\
&+ \frac{\sigma^3 \gamma_1}{n\bar{w}3!} \left[\beta \sum_{k=1}^n \exp(\beta x_k) + (3/\sum_{j=1}^n x_j^2) \sum_{k=1}^n x_k \exp(\beta x_k) \right] \\
&+ \frac{\sigma^4 (\gamma_2 + 3)}{n\bar{w}4!} \left[\beta \sum_{k=1}^n \exp(\beta x_k) + (4/\sum_{j=1}^n x_j^2) \sum_{k=1}^n x_k \exp(\beta x_k) \right]
\end{aligned}$$

As $\bar{w} > 0$, the signing of the bias expressions in Corollaries 3 and 4 is the same as for that in Corollary 1.

4. Numerical illustration

We illustrate the results in Corollaries 3 and 4 numerically.³ The data for the (fixed) regressor are generated either as standard normal, or as uniform on (0,1). Values of $n = 10, 25$, and 100 ; and $\sigma = 0.05, 0.1$, and 0.2 were considered. The *percentage* bias of the OLS marginal effect estimator is the same for both the log-lin and log-log models, and is invariant to the value of β . The distributions used for u are uniform on $(-1,1)$, standard normal, logistic, Laplace, and Student-t with 5 degrees of freedom. The excess kurtosis (γ_2) values for these distributions are $-1.2, 0.0, 1.2, 3.0$, and 6.0 respectively. In the case of the normal regressor and normal u the regression R^2 values are $0.99, 0.98$ and 0.92 when $n = 10$ and $\sigma = 0.05, 0.1$, and 0.2 . The corresponding R^2 values are $0.99, 0.99$ and 0.96 when $n = 100$.

From the results in Table 1 we that, first, the percentage biases for the marginal effect estimators are relatively small, and there is little difference between the results to $O(\sigma^2)$ and those to $O(\sigma^4)$. However, the percentage biases for the marginal effect estimator in our log-lin or log-log model are one or two orders of magnitudes larger than the corresponding results for OLS elasticity estimator in a fully linear model reported by Chen and Giles (2007, p.190). In their application (with $n = 11$), the percentage biases at the sample mean ranged from 0.001% to 0.04% .

Second, these relative biases are extremely stable across different error distributions, and hence across different degrees of excess kurtosis. Third, the percentage biases decrease as the sample size increases to $n = 100$, reflecting the consistency of the OLS coefficient estimator. Finally, these biases are mildly sensitive to the form of the regressor data. Slightly larger percentage biases arise when the regressor is uniform than when it is normal.

5. Conclusions

In this paper we have obtained analytic expressions for the biases of estimators of marginal effects in regression models that have a logarithmically transformed dependent variable. These expressions are based on “small disturbance” asymptotic expansions and do not depend on large sample sizes. Results are obtained for the situations where the regressors are measured in the levels or the logarithms of the original data, and where the marginal effects are evaluated either at a single sample value or at the sample mean. Some limited numerical evaluations suggest that the relative magnitudes of these biases are quite small, which is encouraging news for practitioners (who usually ignore this potential bias issue). In addition, the percentage biases appear to be quite robust to the form of the data and the distribution of the error term, and the results to (σ^2) differ only slightly from those to $O(\sigma^4)$.

Table 1: Percentage biases of marginal effect estimators*

$X:$ $n:$ σ	Normal (0,1)			Uniform (0,1)		
	10	25	100	10	25	100
Uniform errors						
0.05	0.1297 [0.1298]	0.1299 [0.1299]	0.1264 [0.1265]	0.1731 [0.1732]	0.1455 [0.1455]	0.1299 [0.1300]
0.1	0.5203 [0.5211]	0.5234 [0.5242]	0.5045 [0.5053]	0.6898 [0.6912]	0.5882 [0.5892]	0.5195 [0.5203]
0.2	2.0911 [2.1041]	2.1203 [2.1334]	2.0031 [2.0153]	2.7344 [2.7554]	2.3987 [2.4149]	2.0707 [2.0836]
Normal errors						
0.05	0.1330 [0.1331]	0.1317 [0.1318]	0.1257 [0.1258]	0.1751 [0.1752]	0.1460 [0.1461]	0.1298 [0.1299]
0.1	0.5444 [0.5458]	0.5379 [0.5393]	0.4980 [0.4993]	0.7032 [0.7054]	0.5925 [0.5941]	0.5179 [0.5192]
0.2	2.2477 [2.2709]	2.2393 [2.2623]	1.9409 [1.9605]	2.8053 [2.8269]	2.4324 [2.4598]	2.0044 [2.0666]
Logistic errors						
0.05	0.1299 [0.1300]	0.1319 [0.1321]	0.1252 [0.1253]	0.1707 [0.1709]	0.1482 [0.1483]	0.1292 [0.1293]
0.1	0.5210 [0.5229]	0.5376 [0.5395]	0.4908 [0.4926]	0.6684 [0.6714]	0.6067 [0.6091]	0.5096 [0.5115]
0.2	2.0807 [2.1108]	2.1965 [2.2281]	1.8306 [1.8566]	2.5239 [2.5691]	2.4885 [2.5279]	1.9277 [1.9557]
Laplace errors						
0.05	0.1296 [0.1298]	0.1313 [0.1315]	0.1256 [0.1257]	0.1711 [0.1714]	0.1471 [0.1473]	0.1293 [0.1295]
0.1	0.5192 [0.5219]	0.5341 [0.5368]	0.4951 [0.4976]	0.6730 [0.6773]	0.5996 [0.6030]	0.5123 [0.5150]
0.2	2.0761 [2.1190]	2.1896 [2.2346]	1.8887 [1.9270]	2.5814 [2.6474]	2.4628 [2.5185]	1.9751 [2.0161]
Student-t errors						
0.05	0.1297 [0.1300]	0.1310 [0.1313]	0.1257 [0.1259]	0.1717 [0.1721]	0.1468 [0.1471]	0.1295 [0.1297]
0.1	0.5201 [0.5241]	0.5316 [0.5357]	0.4968 [0.5006]	0.6777 [0.6842]	0.5980 [0.6031]	0.5139 [0.5179]
0.2	2.0838 [2.1484]	2.1726 [2.2397]	1.9120 [1.9701]	2.6225 [2.7233]	2.4561 [2.5394]	1.9967 [2.0589]

*Values are to $O(\sigma^2)$. Those in brackets are to $O(\sigma^4)$.

References

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Footnotes

1. Throughout this paper, “log” denotes the natural (base ‘e’) logarithm.
2. If the explanatory variable of interest is log-transformed, but the dependent variable is in levels, there is no issue. In this case the estimated marginal effect depends on the estimated coefficient and the *non-random* observations on the explanatory variable. The properties of the marginal effect estimator follow trivially from those of the coefficient estimator.
3. The EViews program code that was used is available from the author, on request.